IEOR E4602: Quantitative Risk Management (Spring 2015)
Columbia University
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Assignment 8: Due Monday 4^{th} May 2015

Question 1
(a) Let $F(x) := 1 - (\kappa/(\kappa + x))^\alpha$ for $\alpha > 0$, $\kappa > 0$ and $x \geq 0$ denote the CDF of the Pareto distribution. By considering the normalizing sequences $c_n = \kappa n^{1/\alpha}/\alpha$ and $d_n = \kappa n^{1/\alpha} - \kappa$, show that $F \in \text{MDA}(H_\xi)$. What is the value of ξ?

(b) Confirm your result of part (a) by applying a theorem from the lecture notes regarding the Fréchet MDA. In particular, you should specify the function, $L(\cdot)$, that is slowly varying at infinity.

Question 2
Let $Z \sim H_{\xi,\mu,\sigma}$. Show that

$$W := \left(1 + \xi \, \frac{Z - \mu}{\sigma}\right)^{-1/\xi}, \quad \xi \neq 0$$

has an exponential distribution with mean 1. Explain how this might be used to check the GEV model's goodness of fit given data $Z_1, \ldots, Z_n$.

Question 3
Use the threshold exceedance method to estimate $\text{ES}_{.99}$ using the Danish fire data. Compute a 95% confidence interval for your estimate by assuming the maximum likelihood estimates of ξ and β have a bivariate normal distribution and then using Monte-Carlo simulation. Compare your confidence interval with the empirical estimate of $\text{ES}_{.99}$. (This is a very approximate way to construct confidence intervals. There are better ways based on re-parametrization or possibly bootstrapping methods.)

Hint: You can load the data in *R* by installing the *evir* package and then typing *data(danish)* at the *R* prompt. Further useful commands include *out* $\leftarrow$ *gpd(danish,x)* where x is the threshold level that you need to specify, and *riskmeasures(out, c(0.99))*.