IEOR E4602: Quantitative Risk Management (Spring 2016)

Columbia University

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Assignment 5: Due Monday 4^{th} April 2016.

Answer questions 1, 4 and 5 and one other question.

Question 1

In this question you will implement the Black-Litterman framework of Section 2 in the *Asset Allocation and Risk Management* lecture notes. We will assume that X is an $n \times 1$ random vector of excess returns on n securities. (The *Matlab* function *MarkowitzAnalysisAssignment5.m* can easily be adapted to do parts (b) and (c). This function has been posted on *CourseWorks* but you are welcome to use another software package / language if you prefer.)

(a) Write a piece of code to compute μ_{mar} and $\mathbf{\Sigma}_{mar}$.

(b) Let $\mathbf{\Omega} = \mathbf{P}\mathbf{\Sigma}\mathbf{P}'/c$ where c is a scalar controlling the level of confidence that we have in our views. Write a function that plots the efficient frontier for $c = c_1 < c_2 < \cdots < c_m$. Your function should plot the m frontiers on the same figure.

(c) Run your code from part (b) (which of course uses your code from part (a)) with some sample data and views. You could for example, consider just a single view where you believe that the first security will have a greater return than the mean return, π_1, of the prior distribution. (But you should still make your code general enough to accommodate multiple views via the matrix, $\mathbf{P}$.)

How do the efficient frontiers behave as a function of c? Can you explain this behavior?

Question 2

Do Exercise 12 in the *Asset Allocation and Risk Management* lecture notes. (Only instead of convincing yourself, you should convince the grader!)

Question 3

Do Exercise 13 in the *Asset Allocation and Risk Management* lecture notes.

Question 4

Let $\mathbf{R}$ denote an N-dimensional vector of date T log returns. You may assume $\mathbf{R} \sim \text{MVN}(\mu, \mathbf{\Sigma})$ where $\mu_i = 10\%$ for $i = 1, \ldots, N/2$ and $\mu_i = 20\%$ for $i = N/2+1, \ldots, N$. Other parameters are $\sqrt{\mathbf{\Sigma}_{i,i}} = 30\%$ for $i = 1, \ldots, N$ and $Corr(R_i, R_j) = 30\%$ for all $i \neq j$. Finally you should take $N = 10$, $T = .5$ and $r = 3\%$ where r is the annualized continuously compounded risk-free interest rate.

(a) Write a piece of code that simulates M sample vectors $\mathbf{R}_1, \ldots, \mathbf{R}_M$ and uses these sample vectors to solve for the portfolio that maximizes the expected return subject to $\text{CVaR}_{.95} \leq 50\%$.

(b) Investigate the *bias* in the calculated CVaR as a function of M. (For a fixed value of M you can do this by simulating the returns on the portfolio from part (a), estimating the corresponding 95% CVaR and comparing it to 50%.)

(c) Repeat parts (a) and (b) but now with no short-sales constraints on the risky securities. Note that you are still free to borrow at the risk-free rate.

(d) What do you think would be the impact of estimation errors in the portfolio chosen in parts (a) and (c)?

(Note that it was not necessary to assume normality of returns in this question. We could have chosen any distribution, including one that incorporated both subjective and objective views on the market.)

Question 5

Read the paper *"The Devil Is in the Tails: Actuarial Mathematics and the Subprime Mortgage Crisis"* by Donnelly and Embrechts (2009). Then write a program in the language of your choice that replicates Figure 4(c) from Section 6.1 of that paper. By playing around with different values, make sure you understand the sensitivities of the expected tranche losses to the various parameters. *Hint:* See Section 5.1 of the *An Introduction to Copulas* lecture notes!