IEOR E4602: Quantitative Risk Management (Spring 2016)
Columbia University
Instructor: Martin Haugh
Assignment 1: Due Wednesday 3^{rd} February 2016

Question 1

Consider the options data in the *OptionsPivotTables.xls* Excel workbook. We want to consider *joint* stresses of the underlying security price and implied volatility. In particular, we want to consider shifting the underlying security by -20%, -10%, -5%, 0%, $+5\%$, $+10\%$ and $+20\%$, and shifting the implied volatility by -5, -2, 0, $+2$, $+5$, $+10$ and $+20$ volatility points. We therefore have a total of $7 \times 7 = 49$ scenarios.

Write a `VBA` subroutine that takes all the options data in the *OptionsData* worksheet, computes the P&L for each option under each scenario and outputs the results to the same spreadsheet beginning at column *AR*. In completing this task it is a good idea to:

(i) Turn off automatic screen updating at the beginning of your code by using the

Application.ScreenUpdating = FALSE

command. It is a good idea to do this in general as it can significantly speed up the execution of your `VBA` code. At the end of your subroutine you must remember to include an *Application.ScreenUpdating = TRUE* statement.

(ii) Determine the number of positions in the portfolio. This can be done using

NumPositions = Range(ActiveSheet.Range("ABC"), ActiveSheet.Range("ABC").End(xlDown)).Count

in your code. *NumPositions* would be declared as an integer variable at the start of your code and *"ABC"* is a named range for a cell in the first row of the portfolio, i.e., row 5 in this case. Alternatively you could replace *"ABC"* with the cell address *"A5"*.

(iii) You should create an array, *PositionData* say, with dimensions *NumPositions* $\times X$, to contain the data you need from the *OptionsData* worksheet. Here X is the number of fields you require to revalue each position under the various scenarios. This array can be filled using a for-loop to cycle through the *NumPositions* rows in the worksheet. Since the array will need to contain different data-types, you will need to define this array as being of type *Variant*.

(iv) Create a second multi-dimensional array, *ScenarioData* say, in your `VBA` code with dimensions *NumPositions* $\times$ 49. This array will hold the P&L's arising from the 49 scenarios.

(v) Use a for-loop to run through each row in the *PositionData* array and fill the corresponding row of *ScenarioData* with the results from the 49 scenarios.

(vi) Finally, write the contents of the *ScenarioData* array to the *OptionsData* worksheet, beginning at cell AR5.

Once your subroutine has been written and debugged, place a *button* at the top of the *OptionsData* worksheet so that the subroutine will run when the button is clicked. Note also that `VBA` has an easy-to-use debugger that can greatly simplify the process of debugging your code. The *Immediate*, *Locals* and *Watch* windows are very helpful in this regard. These windows can be added to your `VBA` environment via the *Insert* menu.

Note: An additional PDF file, "The Options Portfolio Excel Spreadsheet", has also been posted as part of this Assignment. This file explains the basic mechanics of the *OptionsPivotTables.xls* Excel workbook and also contains references to online sources for learning `VBA` and learning about pivot-tables.

Question 2
(a) Once the P&L's from the 49 scenarios have been calculated and written to the *OptionsData* worksheet, create a *pivot-table* that can display the P&L from one of these scenarios as a function of strike (rows), maturity (columns) and underlying security (page or filter).

(b) Create additional pivot tables for appropriate delta, gamma, vega, volga and vanna numbers. Check to see if the numbers in these pivot tables are consistent with the pivot table in (a) when the underlying security is shifted by -10% and the implied volatilities increase by $+10$ volatility points.

(c) Suppose you want to create a pivot-table to view the P&L data as a function of the shift in the underlying (rows), the shift in the implied volatilities (columns) and the underlying security (page or filter). Is it possible to do this given the way we have organized the data in the *OptionsData* worksheet? If yes, do it. If not, explain briefly what design changes you would need to have made in Question 1 so that it would be possible to create such a pivot-table.

Note: There are two additional worksheets in the *OptionsPivotTables.xls* Excel workbook that contain pivot tables based on the data in cells $A4$ to $AQ87$ of the *OptionsData* worksheet. You can play with these pivot tables if you wish to get a better feeling for how pivot tables work. But you may also want to consult one of the tutorials listed in the "The Options Portfolio Excel Spreadsheet" PDF file which has been posted as part of the Assignment.

Question 3
(a) Do you think parallel shifts in the volatility surface represent reasonable scenarios? Can you think of a better set of scenarios for representing shifts in the implied volatility surface? Put another way, suppose a crisis occurs tomorrow and all implied volatilities increase. Do you think near-term volatilities will increase more or less than long-term implied volatilities? (Those of you taking the Experimental Finance course could answer this question by examining the empirical behavior of the implied volatility surface. This question is designed to make you aware of the fact that in designing realistic scenarios you ought to be aware how

risk factors typically move in times of crisis.)

(b) Suppose an options portfolio is delta, gamma, vega and vanna neutral. Suppose also that the options portfolio is designed to be very *short skew*, that is the portfolio will benefit if the skew flattens. Do you think scenarios involving shifts in the underlying security and implied volatility surface will reveal the full risk in the portfolio? Explain your answer. What sort of additional scenarios might you want to include? What issues arise when designing these additional scenarios? (This question should impress on you the importance of considering the trading strategy / portfolio composition when designing scenarios. A more basic example occurs if you consider a market-neutral portfolio of stocks. Then just shifting the underlying securities by uniform amounts will not reveal any risk in the portfolio. Likewise, a fixed income portfolio that has a zero duration will reveal very little risk if it is only subjected to stresses based on parallel shifts in the yield curve. Such a portfolio might still be very risky if, for example, it is exposed to changes in the slope of the yield curve or other risk factors.)

Question 4
Compute the VaR and Expected-Shortfall (ES) for (i) the normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 0.3 \times 10000/\sqrt{250}$ and (ii) the t_4 distribution with the same values of μ and σ. (Note that if we assume that the horizon is $\Delta = 1$ day, then a value of $\sigma = .3/\sqrt{250}$ corresponds to an annual volatility of approx 30%. The value of 250 corresponds to the fact that there are approx. 250 trading days in a calendar year. The multiplier of 10000 is there simply to make the numbers more readable.) You should compute the VaR and ES for the following values of α:

$$0.90,\ 0.95,\ 0.975,\ 0.99,\ 0.995,\ 0.999,\ 0.9999,\ 0.99999,\ 0.999999.$$

What do you notice? Now compute the ES to VaR ratio for each value of α. You should report all your results in a table using the same format as Figure 2 of the lecture notes.

Question 5 (Requires `R`)
This question is based on material from Sections 19.2 and 19.3 of Ruppert and Matteson's *Statistics and Data Analysis for Financial Engineering*. (Feel free to look at those sections but you should try and figure out the details yourself rather than just coping the code there.) First install *R*'s *Ecdat* package from CRAN and then load the package by including the line "library(Ecdat)" at the beginning of your *R* session, script or function. We will be using the SP500 dataframe from the *Ecdat* package. This dataframe contains returns on the S&P500 index between 1981 and 1991.

(a) Suppose you hold a $\$1m$ position in the S&P500. Use the return data in the the SP500 dataframe to estimate the 95% VaR and expected shortfall (ES) of your position.

(b) Use *R*'s *fitdistr* function to fit a t-distribution to the return data. Now use this fitted distribution to again estimate the 95% VaR and ES of your position. Compare your answer to your answer from part (a).

(c) This part is entirely **optional** as you are not expected or required to know anything about *bootstrap* methods for this course. But I include the question here nonetheless as it is in general always a good idea to have an idea of how much uncertainty there is in your estimators.

Compute a 90% bootstrap confidence interval for your estimators from part (a). This can be done as follows. Suppose there are n return observations in the SP500 dataframe.

Step 1: Sample *with* replacement n returns from the SP500 data.

Step 2: Estimate the VaR and expected shortfall for this new sample using the method of parts (a).

Step 3: Repeats steps 1 and 2 M times to obtain M estimates of the VaR and ES.

The 5% and 95% quantiles of the bootstrap estimates give you approximate 90% confidence intervals for your VaR and ES estimators. What do you notice? (Note that we could also do this for the parametric estimators of part (b). Feel free to do so if you wish!)

Finally, note that this question focuses entirely on the *unconditional* loss distribution, which is generally a bad idea in practice. (Time series methods can be used to estimate conditional loss distributions. If time permits we will discuss these methods at the end of the course.)