IEOR E4703 Monte-Carlo Simulation
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Due: 12.55pm Tuesday 28 March 2017

Assignment 5

1. Reproduce Figure 1 in the *Simulating Stochastic Differential Equations* lecture notes. Run your code multiple times. What do you notice?

2. Reproduce Figure 2 in the *Simulating Stochastic Differential Equations* lecture notes. (You can ignore the second-order scheme whose details were not provided in the lecture notes.) Repeat but this time simulate $\log S_t$ instead of S_t. (You can actually do everything in the same function and therefore plot four error estimates as a function of n, the number of time steps.)

3. Consider an n^{th}-to-default credit derivative that matures at time T and has N underlying *credits*. The payoff of this security occurs at time T and is given by

$$\text{Payoff} = \sum_{i=1}^{N} m_i \, 1_{\{\tau_i \leq T \text{ and } \tau_i = \tau_{(n)}\}}$$

where τ_i is the default time of the i^{th} credit and $\tau_{(j)}$ is the time of the j^{th} default times (from the underlying N credits). Risk-neutral pricing (with the cash account as numeraire and with a fixed risk-free rate, r) therefore yields

$$\theta := \mathbb{E}_0^Q \left[e^{-rT} \sum_{i=1}^{N} m_i \, 1_{\{\tau_i \leq T \text{ and } \tau_i = \tau_{(n)}\}} \right]$$

as the arbitrage-free price of the security. The state of the economy is modeled via a process X_t and the default intensity for the i^{th} credit is given by $\lambda_t^i := g_i(X_t)$. Conditional on the process X_t, the default processes for the N names are independent non-homogeneous Poisson processes. That is, we are modeling the default of each credit as the first arrival in a Cox process. Moreover conditional on the path of X_t, these processes are independent. Explain carefully how you would estimate estimate θ using Monte-Carlo simulation.

4. **(Pricing Knockout Put Options in a GBM World)**

   (a) Write a function to compute the price of a knockout put option with strike K and barrier $H < K$ in the Black-Scholes framework. (You can search online or in one of your textbooks to find the appropriate formula.) The specific parameters are: initial underlying price $S_0 = 100$, strike $K = 90$, maturity $T = .5$ years, dividend yield $q = 0$, risk-free rate $r = 1\%$, volatility $\sigma = 50\%$ and barrier $H = 80$.

   (b) Write a function that simulates an Euler scheme for S_t and estimates the price of your option.

(c) Extend your function from part (b) so that your scheme also simulates the minimum of the stock price in each discretized interval. You can do this using a suitably adjusted version of formula (24) from the *Simulating Stochastic Differential Equations* lecture notes to simulate the minimum. (Note that in Section 5.2 of the lecture notes, we assumed the barrier was greater than S_0. In this question the barrier is less than the strike which is in turn less than S_0.)

(d) Now run your function to compute two price estimates of the option for $n = 10, 50, 100, 250, 500$ and 1000 time steps and 4 million sample paths. Plot the absolute error in the estimated price on a $\log - \log$ scale as a function of n. Which scheme is superior?

(e) What type of discretization error would be present if we simulated $\log S_t$ instead of S_t? Justify your answer.

(f) From a simulation point of view, do you think it is easier to price a *continuously* monitored barrier option or a *discretely* monitored barrier option?

5. **(Pricing Knockout Put Options Continued)**

(a) Do Exercise 8 in the *Simulating Stochastic Differential Equations* lecture notes.

(b) Do Exercise 9 in the *Simulating Stochastic Differential Equations* lecture notes.