Foundations of Financial Engineering

Incentive Problems in Corporate Finance

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Incentive Problems in Corporate Finance

Recall Merton's structural lattice model of firm default:

- $V_0 = 1,000$, $\mu = 15\%$ and $\sigma = 25\%$.
- $T = 7$ years $\#$ of time periods $= 7$.
- $r = 5\%$
- Face value of debt $= 800$ and coupon $= 0$.

Firm Price Lattice

							6940.6
						5262.6	3990.2
					3990.2	3025.5	2294.0
				3025.5	2294.0	1739.4	1318.9
			2294.0	1739.4	1318.9	1000.0	758.2*
		1739.4	1318.9	1000.0	758.2	574.9	435.9*
	1318.9	1000.0	758.2	574.9	435.9	330.5	250.6*
1000.0	758.2	574.9	435.9	330.5	250.6	190.0	144.1*
t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7

Equity and debt values are 499.7 and 500.3, respectively:

Equity Lattice

							6140.6
						4501.6	3190.2
					3266.4	2264.5	1494.0
				2336.9	1570.2	978.4	518.9
			1640.8	1054.7	603.6	258.0	0.0
		1127.8	687.1	358.4	128.3	0.0	0.0
	758.6	435.7	207.1	63.8	0.0	0.0	0.0
499.7	269.9	117.4	31.7	0.0	0.0	0.0	0.0
t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7

Debt Lattice

							800.0
						761.0	800.0
					723.9	761.0	800.0
				688.6	723.9	761.0	800.0
			653.2	684.7	715.3	742.0	758.2*
		611.6	631.7	641.6	630.0	574.9	435.9*
	560.3	564.3	551.1	511.1	435.9	330.5	250.6*
500.3	488.3	457.5	404.2	330.5	250.6	190.0	144.1*
t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7

Turning Down Good Investments

Suppose the firm is offered a great(!) investment opportunity:

Fair value of investment is $X = 100$ but cost to firm will only be 90.

But firm has no cash available and would therefore have to raise the 90 from current equity holders.

Question: Will the equity holders invest?

Turning Down Good Investments

Can model this situation by first adding X to the initial value of the firm and computing the resulting firm-value lattice:

Firm Value Lattice

							7634.7
						5788.8	4389.3
					4389.3	3328.1	2523.4
				3328.1	2523.4	1913.3	1450.7
			2523.4	1913.3	1450.7	1100.0	834.1
		1913.3	1450.7	1100.0	834.1	632.4	479.5*
	1450.7	1100.0	834.1	632.4	479.5	363.6	275.7*
1100.0	834.1	632.4	479.5	363.6	275.7	209.0	158.5*
t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7

Turning Down Good Investments

Can then compute the equity lattice in the usual manner:

Equity Lattice

							6834.7
						5027.8	3589.3
					3665.4	2567.1	1723.4
				2639.5	1799.6	1152.4	650.7
			1868.4	1224.8	726.9	339.0	34.1
		1296.5	809.4	441.4	176.2	16.9	0.0
	881.1	520.9	261.0	91.5	8.4	0.0	0.0
586.9	327.7	151.3	47.4	4.2	0.0	0.0	0.0
t = 0	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6	t = 7

Note that equity value is now 586.9

- an increase of $586.9 - 499.7 = 87.2$ dollars
- but less than the 90 required to make the investment.

Question: What has happened?

Taking on Bad Investments

Other incentive problems can arise:

Suppose fair value of investment is again 100 but now it costs 110.

Clearly this is a bad investment and should not be made.

But for the equity-holders it may actually be rational to invest if the investment increases the volatility of the firm

- recall equity-holders own a call option on the value of the firm
- and the value of an option increases with volatility.

So possible that increase in equity value due to the increase in volatility will exceed the decrease in equity value due to poor quality of investment

- then it makes sense for equity holders (who control the firm) to invest.

Question: How might you model this situation using Merton's structural model?

Foundations of Financial Engineering

Securitization and CDO's

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Collateralized Debt Obligations (CDO's)

Want to find the expected losses in a simple 1-period CDO with the following characteristics:

- The maturity is 1 year.
- There are $N = 125$ bonds in the reference portfolio.
- Each bond pays a coupon of one unit after 1 year if it has not defaulted.
- The recovery rate on each defaulted bond is zero.
- There are 3 tranches of interest:

1. The equity tranche with attachment points: 0-3 defaults
2. The mezzanine tranche with attachment points: 4-6 defaults
3. The senior tranche with attachment points: 7-125 defaults.

Assume probability, q, of defaulting within 1 year is identical across all bonds.

Collateralized Debt Obligations (CDO's)

X_i is the normalized asset value of the i^{th} credit and we assume

$$X_i = \sqrt{\rho}M + \sqrt{1-\rho}\, Z_i \tag{1}$$

where $M, Z_1, \ldots, Z_N$ are IID normal random variables

- note correlation between each pair of asset values is identical.

We assume also that i^{th} credit defaults if $X_i \leq \bar{x}_i$.

Since probability, q, of default is identical across all bonds must therefore have

$$\bar{x}_1 = \cdots \bar{x}_N = \Phi^{-1}(q). \tag{2}$$

It now follows from (1) and (2) that

$$\begin{aligned} \mathsf{P}(i \text{ defaults} \,|\, M) &= \mathsf{P}(X_i \leq \bar{x}_i \,|\, M) \\ &= \mathsf{P}(\sqrt{\rho}M + \sqrt{1-\rho}\, Z_i \leq \Phi^{-1}(q) \,|\, M) \\ &= \mathsf{P}\left(Z_i \leq \frac{\Phi^{-1}(q) - \sqrt{\rho}M}{\sqrt{1-\rho}} \,\middle|\, M \right). \end{aligned}$$

Collateralized Debt Obligations (CDO's)

Therefore conditional on M, the total number of defaults is $\text{Bin}(N, q_M)$ where

$$q_M := \Phi\left(\frac{\Phi^{-1}(q) - \sqrt{\rho}M}{\sqrt{1-\rho}}\right).$$

That is,

$$p(k \mid M) = \binom{N}{k} q_M^k (1 - q_M)^{N-k}.$$

Unconditional probabilities computed by numerically integrating the binomial probabilities with respect to M so that

$$\mathsf{P}(k \text{ defaults}) = \int_{-\infty}^{\infty} p(k \mid M)\phi(M)\, dM.$$

Can now compute expected (risk-neutral) loss on each of the three tranches:

Collateralized Debt Obligations (CDO's)

$$
\begin{aligned}
\mathsf{E}_0^{\mathbb{Q}}\left[\text{Equity tranche loss}\right] &= 3 \times \mathsf{P}(3 \text{ or more defaults}) + \sum_{k=1}^{2} k\, \mathsf{P}(k \text{ defaults}) \\
\mathsf{E}_0^{\mathbb{Q}}\left[\text{Mezz tranche loss}\right] &= 3 \times \mathsf{P}(6 \text{ or more defaults}) + \sum_{k=1}^{2} k\, \mathsf{P}(k+3 \text{ defaults}) \\
\mathsf{E}_0^{\mathbb{Q}}\left[\text{Senior tranche loss}\right] &= \sum_{k=1}^{119} k\, \mathsf{P}(k+6 \text{ defaults}).
\end{aligned}
$$

Collateralized Debt Obligations (CDO's)

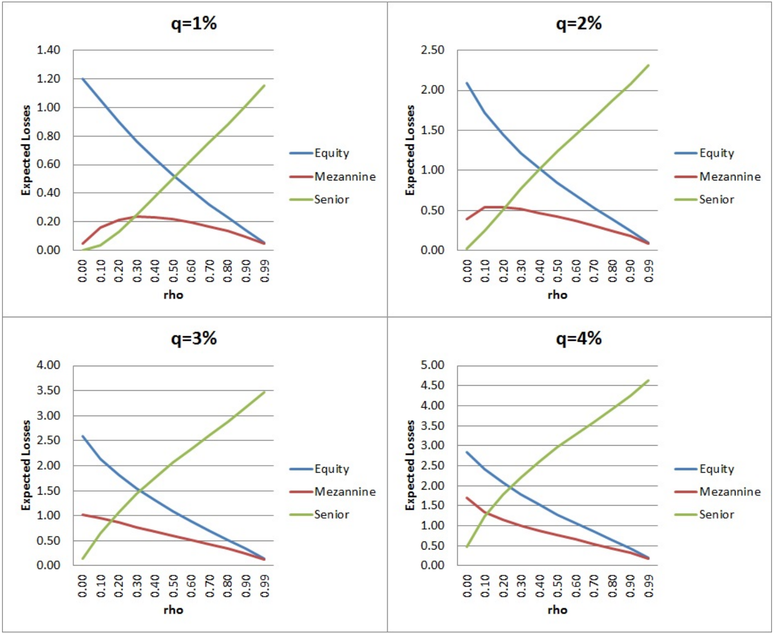
Regardless of the individual default probability, q, and correlation, ρ, we have:

$$\mathsf{E}_0^{\mathbb{Q}}\left[\%\ \text{Equity tranche loss}\right] \geq \mathsf{E}_0^{\mathbb{Q}}\left[\%\ \text{Mezz tranche loss}\right] \geq \mathsf{E}_0^{\mathbb{Q}}\left[\%\ \text{Senior tranche loss}\right].$$

Also note that expected equity tranche loss always decreasing in ρ.

Expected mezzanine tranche loss often relatively insensitive to ρ.

Expected senior tranche loss (with upper attachment point of 100%) always increasing in ρ.



Expected Tranche Losses As a Function of ρ

Collateralized Debt Obligations (CDO's)

Question: How does the total expected loss in the portfolio vary with ρ?

The dependence structure we used in (1) to link the default events of the various bonds is the famous Gaussian-copula model.

In practice CDO's are multi-period securities and can be cash or synthetic CDO's.

Foundations of Financial Engineering

Securitization and MBS's

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Introduction to Mortgage-Backed Securities

Consider a standard level-payment mortgage with initial principal $M_0 := M$.

In each of n periods a payment of size B dollars is made

- after n periods all principal and interest will have been paid
- each payment, B, therefore pays both interest and some of the principal
- such a mortgage is said to be fully amortizing.

If we assume the coupon rate is c per period then can solve for B as follows.

Let M_k denote principal remaining after the k^{th} period. Then $M_n = 0$ and

$$M_k \ = \ (1+c)M_{k-1} - B \quad \text{for} \ \ k = 1, 2, \ldots, n. \tag{3}$$

Can iterate (3) to obtain

$$\begin{aligned} M_k &= (1+c)^k M_0 - B \sum_{p=0}^{k-1} (1+c)^p \\ &= (1+c)^k M_0 - B \left[\frac{(1+c)^k - 1}{c} \right]. \end{aligned} \tag{4}$$

But $M_n = 0$ and so

$$B = \frac{c(1+c)^n M_0}{(1+c)^n - 1}. \tag{5}$$

Can now substitute (5) back into (4) and obtain

$$M_k = M_0 \, \frac{(1+c)^n - (1+c)^k}{(1+c)^n - 1}.$$

Introduction to Mortgage-Backed Securities

Assume a deterministic world with no possibility of defaults or prepayments.

If $r =$ risk-free interest rate per period, then fair mortgage value is

$$\begin{aligned} F_0 &= \sum_{k=1}^{n} \frac{B}{(1+r)^k} \\ &= \frac{c(1+c)^n}{(1+c)^n - 1} \times \frac{(1+r)^n - 1}{r(1+r)^n} \, M_0. \end{aligned} \qquad (6)$$

If $r = c$ then (6) immediately implies $F_0 = M_0$, as expected.

But in general have $r < c$ to account for possibility of default, prepayment, servicing fees, bank profits etc.

Scheduled Principal and Interest Payments

We know M_{k-1} so can compute the interest:

$$I_k \;:=\; cM_{k-1} \tag{7}$$

that would be due in the next period, i.e. period k.

Also means can interpret the k^{th} payment as paying

$$P_k :=\; B - cM_{k-1} \tag{8}$$

of the remaining principal, M_{k-1}.

In any time period, k, can therefore easily break down the payment B into a scheduled principal payment, P_k, and a scheduled interest payment, I_k.

Can take a large pool of these mortgages and use this observation to construct interest-only (IO) and principal-only (PO) mortgage-backed securities (MBS).

There are many other classes of MBS including for example sequential CMO's (collateralized mortgage obligations), PAC CMO's etc.

Prepayment Risk

In practice there is in fact uncertainty in interest and principal payments.

Due to possible default by mortgage holders and possibility of prepayments

- prepayments are payments in excess of scheduled principal payments.

Many mortgages in the US allow prepayment and there are many possible reasons for doing so including default, better refinancing rates, moving home etc.

Prepayment modeling therefore an important feature of pricing residential MBS

- indeed the value of some MBS's very dependent on prepayment behavior.

Prepayment Risk

Question: What do you think happens to the value of a PO MBS (relative to a regular fixed income security) when interest rates (i) increase (ii) decrease?

Question: What do you think happens to the value of an IO MBS (relative to a regular fixed income security) when interest rates (i) increase (ii) decrease?

Foundations of Financial Engineering

Risk Management: the Greeks and Scenario Analysis

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Factor Sensitivity Measures or the "Greeks"

A factor sensitivity measure gives the change in value of a portfolio for a given change in the "risk factors".

e.g. Greeks for an option portfolio or duration / convexity of a bond portfolio

- often used to set position limits on trading desks and portfolios.

Consider an option that is written on an underlying security with price process S_t. Assume option price, C, is a function of only S_t and implied volatility, σ_t.

Then Taylor's Theorem yields

$$\begin{aligned} C(S+\Delta S, \sigma+\Delta\sigma) &\approx C(S,\sigma) + \Delta S \frac{\partial C}{\partial S} + \frac{1}{2}(\Delta S)^2 \frac{\partial^2 C}{\partial S^2} + \Delta\sigma \frac{\partial C}{\partial \sigma} \\ &= C(S,\sigma) \;+\; \Delta S\,\delta \;+\; \frac{1}{2}(\Delta S)^2\,\Gamma \;+\; \Delta\sigma\,\text{vega}. \end{aligned}$$

– have omitted dependence of various quantities on t. Therefore obtain

$$\begin{aligned} \text{P\&L} &\approx \delta\Delta S \;+\; \frac{\Gamma}{2}(\Delta S)^2 \;+\; \text{vega}\;\Delta\sigma \\ &= \text{delta P\&L} \;+\; \text{gamma P\&L} \;+\; \text{vega P\&L}\;. \end{aligned} \qquad (9)$$

Factor Sensitivity Measures or the "Greeks"

When $\Delta\sigma = 0$, obtain obtain the well-known delta-gamma approximation

- often used in historical Value-at-Risk (VaR) calculations.

Can also write (9)

$$
\begin{aligned}
\text{P\&L} \quad &\approx \quad \delta S \left(\frac{\Delta S}{S}\right) + \frac{\Gamma S^2}{2}\left(\frac{\Delta S}{S}\right)^2 + \text{vega } \Delta\sigma \\
&= \quad \text{ESP} \times \text{Return} \ + \ \$ \text{ Gamma} \times \text{Return}^2 \ + \ \text{vega } \Delta\sigma \qquad (10)
\end{aligned}
$$

where ESP denotes the equivalent stock position or "dollar" delta.

Easy to extend this calculation to a portfolio of derivatives.

Factor Sensitivity Measures or the "Greeks"

Important to note that approximations such as (10) are local approximations

- they are based (via Taylor's Theorem) on "small" moves in the risk factors.

These approximations can and indeed do break down in violent markets where changes in the risk factors can be very large.

Scenario Analysis

The scenario approach to risk management defines a number of scenarios where in each scenario the various risk factors are assumed to have moved by some fixed amounts.

e.g. A scenario might assume all stock prices have fallen by 10% and all implied volatilities have increased by 5 percentage points.

Another scenario might assume the same movements but with an additional steepening of the volatility surface.

A scenario for a credit portfolio might assume all credit spreads have increased by some fixed absolute amount, e.g. 100 basis points, or some fixed relative amount, e.g. 10%.

The risk of a portfolio could then be defined as the maximum loss over all of the scenarios that were considered.

A particular advantage of the scenario approach is that it does not depend on probability distributions that are difficult to estimate.

Underlying	SPX Index

Underlying and Volatility Stress Table

Sum of PnL	Vol Stress								
Underlying Stress	-10	-5	-2	-1	0	1	2	5	10
-20	13,938	11,774	10,631	10,277	9,936	9,608	9,293	8,419	7,183
-10	6,109	4,946	4,436	4,291	4,158	4,035	3,922	3,634	3,296
-5	1,831	1,652	1,637	1,643	1,654	1,670	1,689	1,766	1,946
-2	(314)	89	356	447	539	631	723	1,001	1,461
-1	(920)	(338)	15	132	248	363	478	816	1,361
0	(1,463)	(714)	(280)	(139)	0	137	273	668	1,293
1	(1,939)	(1,035)	(527)	(363)	(203)	(45)	110	559	1,259
2	(2,346)	(1,300)	(723)	(539)	(359)	(182)	(9)	489	1,258
5	(3,125)	(1,744)	(1,003)	(769)	(541)	(318)	(102)	518	1,460
10	(2,921)	(1,297)	(423)	(146)	123	385	641	1,372	2,483
20	2,344	3,559	4,272	4,506	4,738	4,967	5,194	5,860	6,919

- P&L for an options portfolio on the S&P 500 under stresses to underlying and implied volatility.
- Can use approximations like (10) to check for internal consistency.

Scenario Analysis

While scenario tables are a valuable source of information there are many potential pit-falls associated with using them. These include:

1. Identifying the relevant risk factors.
2. Identifying "reasonable" shifts for these risk factors.
3. Estimating just how likely each scenario is.

Key role of any risk manager then is to understand what scenarios are plausible and what scenarios are not.

e.g. In a crisis would expect any drop in price of underlying security to be accompanied by a rise in implied volatilities.

Foundations of Financial Engineering

Risk Management and Value-at-Risk

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Loss Distributions

Many risk measures such as value-at-risk (VaR) are based on the loss distribution.

Working with loss distributions makes sense as it contains all relevant information.

A loss distribution implicitly reflects the benefits of netting and diversification.

Moreover, easy to compare loss distribution of a derivatives portfolio with that of a bond or credit portfolio

- at least when same time horizon is under consideration.

But it may be very difficult to estimate the loss distribution!

Consider the series $\mathbf{X_t}$ of risk factor changes and assume they form a stationary time series with stationary distribution $F_{\mathbf{X}}$.

Let $\mathcal{F}_t$ denote all information available in the market at time t, including $\{\mathbf{X_s} : s \leq t\}$.

Loss Distributions

Definition: The unconditional loss distribution is the distribution of L_{t+1} given the time t composition of the portfolio and assuming the CDF of $\mathbf{X_{t+1}}$ is given by $F_{\mathbf{X}}$.

Definition: The conditional loss distribution is the distribution of L_{t+1} given the time t composition of the portfolio and conditional on the information in $\mathcal{F}_t$.

For relatively short horizons, e.g. 1 day or 10 days, then the conditional loss distribution is clearly the appropriate distribution for risk management purposes

- particularly true in periods of high market volatility.

Value-at-Risk

Value-at-Risk (VaR) is the most widely used risk measure in the financial industry.

VaR is calculated using a loss distribution.

Will assume the horizon, Δ, has been fixed, e.g. 1 day or 10 days, and that L represents the loss on the portfolio over the time interval Δ

- will use $F_L(\cdot)$ to denote the CDF of L.

Value-at-Risk

Definition: Let $F : \mathbb{R} \to [0, 1]$ be an arbitrary CDF. Then for $\alpha \in (0, 1)$ the α-quantile of F is defined by

$$q_\alpha(F) \ := \ \inf\{x \in \mathbb{R} \ : \ F(x) \geq \alpha\}.$$

If F continuous and strictly increasing, then $q_\alpha(F) = F^{-1}(\alpha)$.

Definition: Let $\alpha \in (0, 1)$ be some fixed confidence level. Then the VaR of the portfolio loss at the confidence level, α, is given by $\text{VaR}_\alpha := q_\alpha(L)$, the α-quantile of the loss distribution.

The Normal and t Distributions

Normal and t CDFs are both continuous and strictly increasing so easy to calculate their VaR_α.

If $L \sim \mathsf{N}(\mu, \sigma^2)$ then

$$\mathsf{VaR}_\alpha \;=\; \mu + \sigma\Phi^{-1}(\alpha) \quad \text{where } \Phi \text{ is the standard normal CDF.} \tag{11}$$

The Normal and t Distributions

If $L \sim \text{t}(\nu, \mu, \sigma^2)$ so that $(L - \mu)/\sigma$ has a standard t distribution with $\nu > 2$ dof, then

$$\text{VaR}_\alpha \;=\; \mu + \sigma t_\nu^{-1}(\alpha)$$

where t_ν is the CDF for the t distribution with ν degrees-of-freedom.

Weaknesses of Value-at-Risk

VaR has several weaknesses:

1. It attempts to describe the entire loss distribution with a single number
   - so significant information is not captured in VaR
   - this criticism applies to all scalar risk measures.

2. Significant model risk attached to VaR
   - e.g. If loss distribution is heavy-tailed but a light-tailed, e.g., normal distribution is assumed, then VaR_α will be severely underestimated as $\alpha \to 1$.

3. VaR is not sub-additive so it doesn't lend itself to aggregation.

   e.g. Let $L = L_1 + L_2$ be total loss associated with two portfolios. Then

$$q_\alpha(F_L) \;>\; q_\alpha(F_{L_1}) \;+\; q_\alpha(F_{L_1}) \quad \text{is possible.} \tag{12}$$

   – so possibly no diversification benefit when we combine two portfolios.

Foundations of Financial Engineering

Algorithmic Trading

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Algorithmic Trading

Key motivation for algorithmic trading is that investors are no longer price-takers but instead are adversely affected by their trading.

In particular, when they wish to buy (sell) securities the price begins to rise (fall) and so they end up paying more (receiving less) than the original price that prevailed at the beginning of their trading.

Need to account for adverse price moves by modeling temporary and permanent price impact

- magnitude of price impact depends on liquidity of the security.

No strict definition of liquidity but think of a security as being liquid if we can quickly trade large quantities with little price impact, i.e. low slippage.

Optimal Execution of a Single Stock

Consider a simple model for selling a total of X shares over a single day.

In order to execute this trade we break the day into a total of T time periods.

e.g. If we take duration of a time period to be 5 minutes, then will have $T = 78$ time periods if trading on the NYSE.

Let $n_j = \#$ of shares sold in the j^{th} period and let $\mathbf{n} := (n_1, \ldots, n_T)$ denote execution sequence for the T periods.

Set $x_0 = X$ and define $x_k := X - \sum_{j=1}^{k} n_j$ for $k = 1, \ldots, T$

- so x_k denotes number of shares that have yet to be sold after the k^{th} period
- must have $X = \sum_{j=1}^{T} n_j$.

Optimal Execution of a Single Stock

Let S_k denote the pre-trade price for the k^{th} period and let $\hat{S}_k$ denote the realized price-per-share that is obtained for the n_k shares sold at that time.

Assume temporary price impact function, $h(n)$, so that

$$\hat{S}_k = S_k - h(n_k). \tag{13}$$

Model the permanent price impact via the function, $g(n)$, so that

$$S_{k+1} = S_k + \sigma z_k - g(n_k) \tag{14}$$

where the z_k's are IID standard normal random variables and σ is a volatility parameter.

Note that temporary price impact, $h(n_k)$, in period k only affects the realized price in period k.

Optimal Execution of a Single Stock

(13) and (14) imply that total realized revenue of execution strategy satisfies

$$\begin{aligned}\sum_{k=1}^{T}\hat{S}_k n_k &= \sum_{k=1}^{T}\left(S_k - h(n_k)\right) n_k \\ &= \sum_{k=1}^{T}\left(S_1 + \sum_{j=1}^{k-1}\left(\sigma z_j - g(n_j)\right)\right) n_k - \sum_{k=1}^{T} h(n_k) n_k \qquad (15) \\ &= S_1 X + \sigma \sum_{k=1}^{T} z_k x_k - \sum_{k=1}^{T} g(n_k) x_k - \sum_{k=1}^{T} h(n_k) n_k. \qquad (16)\end{aligned}$$

Expected cost of the execution strategy is

$$C(\mathbf{n}) = XS_1 - \sum_{k=1}^{T} g(n_k) x_k - \sum_{k=1}^{T} h(n_k) n_k.$$

Optimal Execution of a Single Stock

The risk, i.e. variance, of the execution strategy is $V(\mathbf{n}) := \sigma^2 \sum_{k=1}^{T} x_k^2$.

Can now formulate optimal execution optimization problem as

$$\min_{\mathbf{n} \geq 0} \; C(\mathbf{n}) + \rho V(\mathbf{n}) \tag{17}$$

where ρ is chosen to tradeoff cost versus risk.

Optimal Execution of a Single Stock

Before solving (17) also need to specify the price impact functions:

- Typical choice for g is a linear impact so that $g(v) = \gamma v$.
- Temporary price impact function typically chosen to be non-linear. One possibility: the Kissel-Glantz function

$$h(n) = a_1 \left(\frac{100\,|n|}{V} \right)^{\beta} + a_2\sigma + a_3$$

$V =$ average daily traded volume and a_i's can be estimated via regression.

(17) can now be solved via standard non-linear optimization methods.

Foundations of Financial Engineering

Algorithmic Trading: Limit Order Books and Dark Pools

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Limit Order Books

The execution model we saw earlier only accounts for one level of overall trading strategy.

Once we have determined $n_k = \#$ of shares to be sold in period k, we must decide how to actually sell these shares.

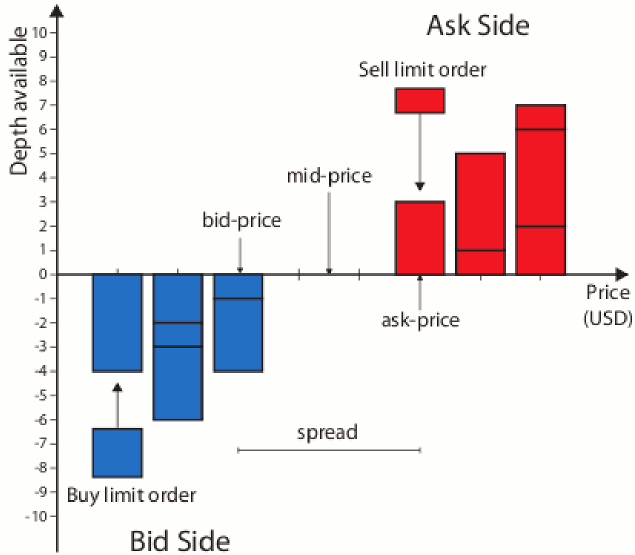
Could use limit orders or market orders executed via the limit-order book but could also send the trades to dark-pools.

Goal at all times is to obtain best execution and prevent other market participants from learning about you and therefore front-running your trades.

A limit-order book is a database that keeps buy orders, i.e. bids, and sell orders, i.e. offers, on a price-time-priority basis.

A limit order has an associated quantity and price at which the order (buy or sell) can be executed.

Limit orders arrive regularly to the order book but the execution of a limit order is uncertain and may never occur.



Taken from the article "Limit Order Books" (2013) by Gould, Porter, Williams, McDonald, Fenn and Howison.

Limit Order Books

This is in contrast to a market order.

e.g. Consider a market order to buy Q shares and let p_o be the best, i.e. lowest, offer price in the limit order book

- let Q_o be the number of shares offered at p_o.

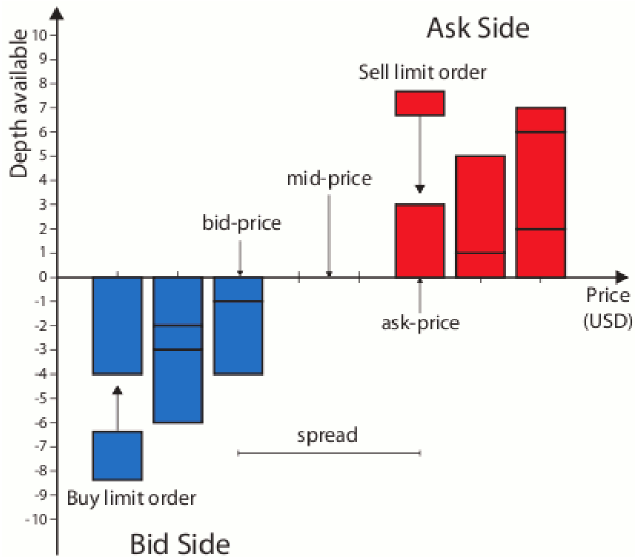
Then the market order will purchase $\min(Q, Q_o)$ shares at price p_o.

If $Q > Q_o$, then remaining $Q - Q_o$ shares will be executed at successively higher offer prices in order-book until all Q shares are purchased.

Market orders to sell shares are transacted similarly by "hitting" the best bids in the order-book.

Advantage of a market order is that execution is guaranteed and immediate.

Disadvantage of a market order is that execution price is not as good and it is necessary to pay (at least) the bid-ask spread.



Taken from the article "Limit Order Books" (2013) by Gould, Porter, Williams, McDonald, Fenn and Howison.

Dark Pools

In contrast to limit-order books, dark pools are trading venues where blocks of shares can be bought or sold without revealing the trade size or identity of agents until trade is filled.

In contrast, regular exchanges (with limit-order books) are called lit pools.

Consequently, dark pool trading hopes to avoid market impact

- but volume of shares executed is uncertain.

There is also some controversy associated with dark pools as they are said to hinder the "price-discovery" process.

Optimal execution in limit-order books and dark pools is currently a very active research area.

Also currently the focus of many market regulators who seek to understand and prevent adverse events such as the flash-crash of May 2010.