IEOR E4706 Foundations of Financial Engineering
Martin Haugh Due: 5pm Friday 23^{rd} September 2016

Assignment 2 (Mandatory)

Examples 1, 2 and 7 in the questions below refer to the examples in the *"Martingale Pricing Theory in Discrete-Time and Discrete-Space Models"* lecture notes.

1. Build a 15-period binomial model whose parameters should be calibrated to a Black-Scholes geometric Brownian motion (GBM) model with: $T = .25$ years, $S_0 = 100$, $r = 2\%$, $\sigma = 30\%$ and a dividend yield of $c = 1\%$. *Hint: Your binomial model should use a value of* $u = 1.0395$. Now answer the following questions:

   (a) Compute the price of an American call option with strike $K = 110$ and maturity $T = .25$ years.

   (b) Compute the price of an American put option with strike $K = 110$ and maturity $T = .25$ years.

   (c) Is it ever optimal to early exercise the put option of part (b)?

   (d) If your answer to part (c) is "Yes", when is the earliest period at which it might be optimal to early exercise?

   (e) Do the call and put option prices of parts (a) and (b) satisfy put-call parity? Why or why not?

2. Referring to Examples 1 and 2, show that

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.3102 \\ 0.4113 \\ 0.2682 \end{bmatrix} + \epsilon \begin{bmatrix} 0.7372 \\ -0.5898 \\ -0.2949 \\ 0.1474 \end{bmatrix}$$

   is also a vector of state prices for any ϵ such that $\pi_i > 0$ for $1 \leq i \leq 4$.

3. What elementary securities are attainable in the model of Example 1? Is this model complete or incomplete? Explain your answer.

4. The single-period model of Example 7 is a complete market. Find the replicating portfolio for each of the elementary securities.

5. **(a)** Referring to Example 7, find a set of risk-neutral probabilities for the case where we take the 2^{nd} security as numeraire. (Recall that the cash account is the 0^{th} security so the 2^{nd} security is the security with price 2.4917 at date $t = 0$.)

   (b) Are these risk-neutral probabilities unique? Explain your answer.

   (c) Would we get the same set of risk-neutral probabilities if we used a different numeraire?