IEOR E4706 Foundations of Financial Engineering
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Due: Wednesday 14^{th} December 2016

Assignment 10

1. **(Incentive Problems in Corporate Finance)**
Consider the incentive problem in Section 1.1 of the *Other Topics in Quantitative Finance* lecture notes where we saw a situation whereby the equity holders would turn down a good investment costing \$90 but with value \$100.

    (a) Suppose instead the current equity-holders raised the cash by issuing \$90 of new *non-voting* equity to *outside* investor. These new equity holders would own $90/(90+499/7) = 15.26\%$ of the firm. Would the original equity holders now vote to make the investment?

    (b) Is there anything wrong with the scenario in part (a)?

    (c) What demands would rational outside investors make in order for them to inject \$90 of new equity into the firm so that the good investment could be made? Would the current equity holders agree to those demands?

    (d) What would have to happen in order for the equity holders to make the new investment?

2. **(Corporate Finance and the Distribution Decision)**
Read the short note *"The Distribution Decision"* (2014, *Journal of Portfolio Management*) by Harold Bierman, Jr. This note discusses the issues regarding a company's *distribution decision*, i.e. what a company should do with its earnings. There are essentially three choices:

    (a) Retention of earnings
    (b) Pay a cash dividend
    (c) Execute a share buyback

    but combinations of these are also possible. The note is short and focuses on the tax implications of the various decisions. This is an important topic in corporate finance and given the very large quantity of share buybacks in recent years, it currently receives a lot of coverage in the financial press. (You don't have to submit anything for this exercise!)

3. **(Mortgage-Backed Securities)**
Consider the interest-only (IO) and principal-only (PO) securities in a deterministic world without prepayments and defaults. These securities have time k cash-flows of $P_k := B - cM_{k-1}$ and $I_k := cM_{k-1}$, respectively, for $k = 1, \ldots, n$ and where M_k (and all other notation) is defined in Section 2.2 of the *Other Topics in Quantitative Finance* lecture notes.

    (a) Compute the present value, V_0, of the PO security.

(b) What happens to V_0 as $n \to \infty$?

(c) Compute the present value, W_0, of the IO security.

(d) Which of the two securities do you think has the longer duration? Justify your answer.

4. **(Adjustable Rate Loans / Mortgages)**
Adjustable-rate loans (ARL) occur very frequently in practice. They are *amortizing* loans – just like many mortgages – except the interest rate is usually tied to some stochastic index. For example, the rate charged might be some T-bill rate plus 250 basis points. The payment in any period is then calculated like a regular amortizing loan where the payment in the next period will be calculated *under the assumption* that the interest-rate will remain constant, e.g. at the *current* T-bill rate + 250 basis points in our example, until then. (Of course the T-bill rate will *not* remain constant so the payments will actually vary from period to period.)

   (a) Construct a 4-period short-rate lattice with initial short-rate $r_0 = 7\%$ and where the short-rate grows by a factor of $u = 1.3$ or falls by a factor of $d = .9$ in each period.

   (b) At each node, $N_{i,j}$, of the lattice determine the *payment rate* per \$100 of principal for a regular amortizing loan initiated at that node, with fixed maturity $t = 5$ periods and interest-rate $r_{i,j} + 2\%$. Note that the payment rate at $t = 5$ will trivially be 100 since the loan initiated then will be due immediately.

   (c) How could you use your lattice to *price*, i.e. determine the value of, the cash-flows associated with an ARL initiated at $t = 0$ and maturity $t = 5$. Note that the cash-flows from the ARL at any node are *path-dependent* since the outstanding principal at that node will be path-dependent.

   Hint: Note the payment due at any node, $N_{i,j}$, is equal to $x_{i,j} \times P$ for some $x_{i,j}$ where P is the outstanding principal from the previous period. The $x_{i,j}$'s are *not* path dependent so explain how to find them at each node for a fixed value of P, say $P = 100$. Then explain how they can be used to determine the value of the ARL.

5. **(Adjustable Rate Loans Ctd: Example 16.5 in Luenberger)**
Denise just graduated from college and has agreed to purchase a new automobile. She is now faced with the decision of how to finance the \$10,000 balance she owes after her down payment. She has decided on a 5-year loan, but is given two choices: (A) a fixed-rate loan at 10% interest or (B) an adjustable-rate loan with interest that at any year is 2 points above the 1-year T-bill rate at the beginning of that year. Currently the T-bill rate is 7%. She wants to know which is the better deal. She makes the assumption that all payments are made annually, starting at the end of the first year.

(a) Compute the value of the fixed-rate and adjustable-rate loans to the bank assuming the short-rate lattice of Question 4 above. (You can assume that the short-rate corresponds to the 1-year T-bill rate.) Given your answers, what should Denise do?

Hint: Use your calculations from parts (b) and (c) of Question 4.

This example comes from Section 16.5 of Luenberger and uses the *levelling* technique to compute the value of the ARL. These loans are clearly very similar to adjustable rate mortgages (ARMs) which were used for a lot of *sub-prime* loans. (These loans often had low, i.e. *teaser*, fixed rates for the first two or three years of the mortgage after which they behaved like ARL's.

(b) Now suppose the adjustable-rate loan is modified by the provision of a cap that guarantees the borrower that the interest rate to be applied will never exceed 11%. What is the value of this loan to the bank? (This is Exercise 16.4 from Luenberger.)